Journal of Applied Mechanics and Technical Physics, Vol. 39, No. 6, 1998

DETERMINATION OF THE STRAIN CHARACTERISTICS OF INTERBLOCK CONTACTS USING ROCK SPECIMEN TESTING RESULTS

L. A. Nazarov and L. A. Nazarova

UDC 539.3

A technique for interpreting rock specimen compression testing data is proposed. It consists of mathematical modeling of the loading of a specimen containing the sliding line L, on which the stresses are continuous and the displacements are discontinuous. At each step of loading the values of a jump of displacements on L are found from the experimentally known stress–strain dependences. The empirical relations describing the strain process on the sliding line are found using these results. Based on Stavrogin's experimental data, we estimate the parameters of the dependence of the peak strength of the interblock contact on the shift from the normal stress.

Introduction. The conclusion that a geophysical medium is of a block structure was made based on an analysis of the deformation processes in objects of different scale levels [1]. This allowed one, in particular, to explain the relatively long lifetime of big tectonic structures despite frequent natural collisions. The mechanism of the latter was associated with a fracture-free repacking of a system of blocks.

These mechanisms also occur at lower scale levels. For example, in biaxial compressive rock tests, Stavrogin and Protosenya [2] obtained data indicating that the specimen preserves the integrity until the strength limit is reached in the constrained conditions. The specimen was then divided into parts, which slid over each other. The typical experimental stress–strain (σ–ε) diagram consists of four parts (Fig. 1) [2, 3]: AB is the elastic section, BC is the nonlinear-elastic section, CD is the descending branch, and DE is the section of beyond-the-limit deformation. Comparing this diagram with the experimental data on the determination of the strain characteristics of the interblock contacts in the tangent direction [4–6], one can conclude that they are in full qualitative agreement.

Revuzhenko and Shemyakin [7] proposed a different mechanism of specimen strain, namely, the penetration of a randomly burnt crack. At the first stage, corresponding to AB, the specimen is single-piece and is deformed elastically; then the sliding line L (Fig. 2), oriented along the site of action of the maximum tangential stress is involved (the section BC), L being in a limiting state; at the third stage (the section CD) the curve L passes to the descending section (the specimen disintegrates) and, finally, the slipping of these parts occurs on the section DE (the residual-strength section on L).

Our interpretation of the rock testing data using the approach of [7, 8] is also motivated by the somewhat incorrect, in our opinion, attempts [9] to assign the experimentally obtained laws of interblock fracture deformation to continual objects, i.e., nonzero-measure elements (for example, in a finite-element analysis), whereas all characteristic features of the behavior of a medium (object) in a beyond-the-limit state are localized in narrow regions, namely, in the planes of sliding.

1. Formulation and Solution of the Problem. In the experiment the specimen was loaded "rigidly" in the vertical direction and "softly" in the horizontal. This process can be modeled as follows. A variable vertical displacement is applied at the upper boundary of the rectangular region G, whose dimensions are l_x and l_z on the x and z axes of the Cartesian coordinate system, the lower boundary does not move in the vertical direction, and the constant horizontal stress is given at the side boundaries (Fig. 2). The curve

Mining Institute, Siberian Division, Russian Academy of Sciences, Novosibirsk 630091. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 39, No. 6, pp. 142–147, November–December, 1998. Original article submitted March 5, 1997.

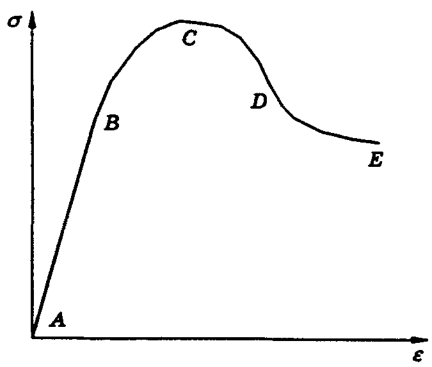

Fig. 1

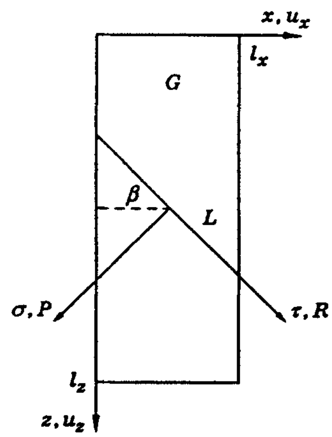

Fig. 2

of sliding L, on which the stresses are continuous and the displacements have a strong discontinuity, passes through the center of the region G at an angle β to the horizontal axis [7, 8]. In modeling the strain process for structured media, e.g., a rock massive, the interblock contacts (discontinuities) are described using precisely these objects [6].

Outside L, the equilibrium equations

$$\sigma_{ij,j} = 0 \tag{1.1}$$

and the Hooke law

$$\sigma_{ij} = 2\mu\Big(\frac{\nu}{1-2\nu}\varepsilon\delta_{ij} + \varepsilon_{ij}\Big) \tag{1.2}$$

are satisfied. Here $\varepsilon_{ij} = 0.5(u_{i,j} + u_{j,i})$, $\varepsilon = \varepsilon_{xx} + \varepsilon_{zz}$, σ_{ij} and ε_{ij} are the stress- and strain-tensor components, $i, j = x, z$, $\mu = 0.5E/(1+\nu)$, E is the Young modulus, and ν is the Poisson ratio; summation is performed over the repeat index.

Also, we formulate the boundary conditions

$$\begin{aligned} &u_z = w_0, \quad \sigma_{xz} = 0 \quad \text{for} \quad z = 0, \\ &u_z = 0, \quad \sigma_{xz} = 0 \quad \text{for} \quad z = l_z, \\ &\sigma_{xx} = \sigma_1, \quad \sigma_{xz} = 0 \quad \text{for} \quad x = 0, l_x, \end{aligned} \tag{1.3}$$

where w_0 is the variable vertical displacement (the load parameter) and σ_1 is the magnitude of the lateral pressure.

In the experiments [2, 3], for various values of σ_1 a series of dependences was obtained:

$$\sigma_2 = \sigma_2(\varepsilon_2), \quad \varepsilon_1 = \varepsilon_1(\varepsilon_2), \tag{1.4}$$

where σ_2 is the reduced stress at the upper edge of the specimen, $\varepsilon_2 = w_0/l_z$, $\varepsilon_1 = u_0/l_x$, ε_1 is the transverse strain, and u_0 is the horizontal displacement.

Thus, we have two characteristics of the loading (1.4) and formulate the following problem: to obtain equations that determine the strain law on L in the form

$$\tau = \tau(R, P), \qquad \sigma = \sigma(R, P)$$

(τ and σ are the tangential and normal stresses, R and P are the slippage and approach of the crack sides) such that the calculation results correspond to the experimental data at each step of loading.

Revuzhenko [8] proposed the variational principle: in the region containing the line of strong discontinuity of displacements, the stress–strain state in which the potential energy of the system W is minimal is realized; note that

$$W = W_e + l_x U/\cos\beta, \tag{1.5}$$

where W_e is the strain energy in the elastic part of the region and $U = U(R, P)$ is the density of energy dissipation on L.

System (1.1)–(1.3) admits the solution in stresses $\sigma_{xx} = \sigma_1$, $\sigma_{zz} = \sigma_2$, and $\sigma_{xz} = 0$; for a plane strain, we have

$$W_e = 0.5A(au_e^2 + 2\nu_* u_e w_e + bw_e^2), \qquad A = E\frac{1-\nu}{(1+\nu)(1-2\nu)}, \qquad \nu_* = \frac{\nu}{1-\nu}, \tag{1.6}$$

where u_e and w_e are the elastic components of the displacements, $a = l_x/l_z$, and $b = l_z/l_x$. The full displacements consist of the elastic displacements and the displacements caused by the continuity violation (Fig. 2):

$$u_0 = u_e + R\cos\beta + P\sin\beta, \qquad w_0 = w_e + R\sin\beta - P\cos\beta. \tag{1.7}$$

Substituting (1.6) and (1.7) into (1.5) and varying with respect to R and P, we obtain the necessary condition for the existence of the minimum of W:

$$r_1 - c_1 R - c_2 P = q\frac{\partial U}{\partial R}, \qquad r_2 - c_2 R - c_3 P = q\frac{\partial U}{\partial P}, \tag{1.8}$$

where $r_1 = m_1 w_0 + m_2 u_0$, $r_2 = m_3 w_0 - m_4 u_0$, $m_1 = a\sin\beta + \nu_*\cos\beta$, $m_2 = b\cos\beta + \nu_*\sin\beta$, $m_3 = a\cos\beta - \nu_*\sin\beta$, $m_4 = b\sin\beta - \nu_*\cos\beta$, $c_1 = a\sin^2\beta + \nu_*\sin 2\beta + b\cos^2\beta$, $c_2 = 0.5(a-b)\sin 2\beta + \nu_*\cos 2\beta$, $c_3 = a\cos^2\beta - \nu_*\sin 2\beta + b\sin^2\beta$, $q = l_x/A\cos\beta$. Taking into account the continuity of the stress field upon passage through L [8], i.e., $\tau = \tau_n$ and $\sigma = \sigma_n$, where $\tau_n = 0.5(\sigma_1 - \sigma_2)\sin 2\beta$ and $\sigma_n = \sigma_1\sin^2\beta + \sigma_2\cos^2\beta$, and the fact that $\tau = \partial U/\partial R$ and $\sigma = \partial U/\partial P$, from (1.8) we obtain the system of equations for the determination of R and P:

$$c_1 R + c_2 P = r_1 - q\tau_n, \qquad c_2 R + c_3 P = r_2 - q\sigma_n. \tag{1.9}$$

The determinant of the system is $c_1 c_3 - c_2^2 = 1 - \nu_*^2 > 0$, and therefore the solution of (1.9)

$$R = \Big(1 + a\frac{\nu_*}{1-\nu_*^2}\sin 2\beta\Big)w_0\sin\beta + \Big(1 + \frac{2\nu_*^2}{1-\nu_*^2}\sin^2\beta\Big)u_0\cos\beta - q_1 s_1,$$

$$P = w_0\cos\beta - u_0\sin\beta - q_1 s_2, \qquad s_1 = m_4\sigma_1\sin\beta - m_3\sigma_2\cos\beta, \tag{1.10}$$

$$s_2 = m_2\sigma_1\sin\beta + m_1\sigma_2\cos\beta, \qquad q_1 = \frac{l_x}{\cos\beta}\,\frac{1-\nu^2}{E}$$

always exists. For (1.10) to describe the state of stable equilibrium, the following sufficient conditions of existence of the minimum of W should be satisfied:

$$\frac{\partial\tau}{\partial R} > -\frac{c_1}{q}, \quad \Big(c_1 + q\frac{\partial\tau}{\partial R}\Big)\Big(c_3 + q\frac{\partial\sigma}{\partial P}\Big) > \Big(c_2 + q\frac{\partial\tau}{\partial P}\Big)^2,$$

which can be checked after solving (1.9).

For various values of the lateral pressure σ_1, the calculations yield a series of dependences

$$\sigma = \sigma(R(w_0), P(w_0)), \tag{1.11}$$

$$\tau = \tau(R(w_0), P(w_0)), \tag{1.12}$$

which were analyzed.

2. Analysis of Results. The calculations were carried out for granite and sandstone specimens, the physical properties of which are given in Table 1. The specimen sizes are $l_x = 0.03$ m and $l_z = 0.08$ m. In all experiments, the angle of declination of the discontinuity was almost constant, $\beta = 45°$, and precisely this value was used in our calculations. For the granite specimen, solid curves 1–4 in Fig. 3 indicate the calculated dependences (1.12) for $\sigma_1 = 10$, 25, 50, and 100 MPa, respectively.

TABLE 1

Specimen	E, GPa	ν	σ_*, MPa
Sandstone	30	0.17	500
Granite	55	0.12	950

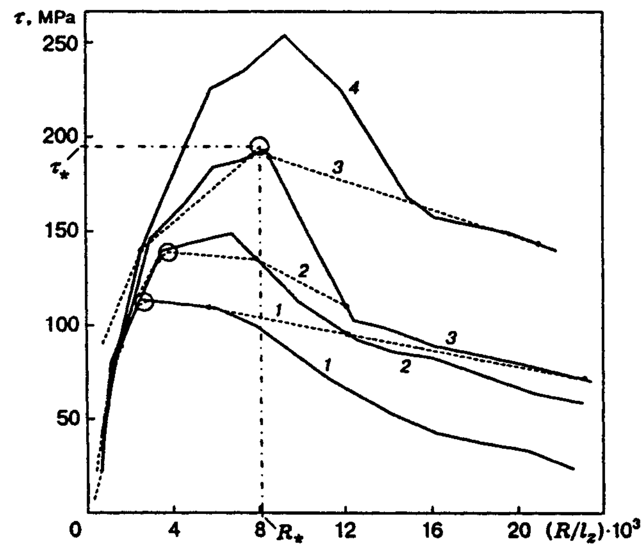

Fig. 3

The following form of the equations describing the law of deformation of interblock contacts was adopted [5]:

$$\sigma = S(P, \tau = 0); \tag{2.1}$$

$$\tau = T(R, \sigma = \text{const}). \tag{2.2}$$

The functions S and T are determined from deformation experiments on rock specimens containing real violations of the continuity. The basic characteristics of T are as follows: τ_* is the peak (maximum) shear strength, R_* is the ultimate slippage (Fig. 3), and $K_t = \partial T/\partial R$ is the tangential stiffness.

A dependence similar to (2.2) can be derived from the calculated data. The value of $\sigma = \sigma^m$ is fixed, and $R^m = \sigma^{-1}(\sigma^m)$ is found from (1.11) and $\tau^m = \tau(R^m, P(R^m))$ from (1.12). The results of this procedure for $\sigma^m = 120$, 160, and 240 MPa are shown in Fig. 3 by dashed curves 1–3, respectively. The scarce experimental data (a large step in σ_1) did not allow one to determine the more detailed form of the function T. Nevertheless, the resulting dependences of the tangential stress on the slippage have characteristic features [4, 5]: an increase in τ_*, R_*, and K_t (for small R) with increasing σ and a descending branch for $R \geqslant R_*$.

Unfortunately, this analysis is unable to determine the form of the function S in (2.1), because the condition $\tau > 0$ is always satisfied during loading.

The peak strength τ_* is one of the basic parameters of the equations describing the empirical law of deformation of interblock contacts, which is used to evaluate the stability of the structured massive. Formulas for estimating τ_* were proposed. We analyze three of them:

- The Coulomb–Moore formula [6]

$$\tau_* = \sigma \tan \alpha_* + \tau_c \tag{2.3}$$

(α_* is the analog of the angle of internal friction and τ_c is the cohesion);

TABLE 2

Sandstone			Granite		
σ_1, MPa	τ_*, MPa	σ, MPa	σ_1, MPa	τ_*, MPa	σ, MPa
0	71.0	71.0	0.5	113.7	114.2
5	87.5	92.5	10	113.5	123.5
10	90.0	100.0	25	149.0	174.0
25	113.5	138.5	50	193.0	243.0
50	151.0	201.0	100	254.0	354.0
100	169.0	269.0	150	371.0	521.0

TABLE 3

Specimen	α_*, deg	τ_c, MPa	r,%	α_*, deg	f	r,%	α_*, deg	h	r,%
	Formula (2.3)			Formula (2.4)			Formula (2.5)		
Sandstone	26.8	40.3	4.7	27.3	9.3	3.2	47.8	0.518	2.0
Granite	32.3	38.0	2.0	30.6	6.1	4.0	44.0	0.324	6.7

- The Barton formula [5]

$$\tau_* = \sigma \tan\left(\alpha_* + f \ln \frac{\sigma_*}{\sigma}\right); \tag{2.4}$$

- The Stephansson formula [10]

$$\tau_* = \sigma \tan\left(\alpha_*(1 - \sigma/\sigma_*)^h\right), \tag{2.5}$$

where σ_* may be treated as an analog of the rock strength in the constrained conditions. The values of σ_* are listed in Table 1.

We consider that the points on the curves of Fig. 3, where τ_* is maximum, correspond to the strength limit τ. The quantity σ was determined for the same value of R. The results of this analysis of the calculated data are shown in Table 2, and their processing by the least-squares method allowed us to find the constants in (2.3)–(2.5) (Table 3). Formulas (2.3)–(2.5) well describe the experimental and calculated data: the relative error r does not exceed 7%. Only (2.5) for small values of σ is not suited for describing the experimental data, because $\tau_* > \sigma$ for sandstone.

It is noteworthy that the values of the cohesion in formula (2.3) are close to $\tau_* = 10$–30 MPa [6]. The open circles in Fig. 3 correspond to the points (R_*^m, τ_*^m) whose ordinate is calculated by (2.3) with allowance for the data of Table 3 and whose abscissa is obtained by interpolation using relation (1.12) and the value of τ_*^m.

The results point to the possibility of estimating the characteristics of the newly formed (as a result of natural or industrial collisions) discontinuities without direct experiments on the standard equipment for specimen testing, which is important, because full-scale experiments are extremely expensive [11], and it is often impossible to extract the discontinuity-containing specimens.

The experimental results can be extrapolated to the extended discontinuities using the scale effect data [12].

The authors are grateful to Academician E. I. Shemyakin for the suggested idea and a discussion of the results.

This work was supported by the Russian Foundation for Fundamental Research (Grant No. 95-05-15604).

REFERENCES

1. M. A. Sadovskii, L. G. Bolkhovitinov, and V. F. Pisarenko, *Deformation of the Medium and the Seismic Process* [in Russian], Nauka, Moscow (1987).
2. A. N. Stavrogin and A. G. Protosenya, *Mechanics of Rock Deformation and Fracture* [in Russian], Nedra, Moscow (1992).
3. A. N. Stavrogin and B. G. Tarasov, "Some results obtained with high-stiffness rock testing systems," *News J. Int. Soc. Rock Mech.*, **3**, No. 2, 7–19 (1995).
4. N. R. Barton, "Deformation phenomena in jointed rock," *Geotechnique*, **36**, No. 2, 147–167 (1986).
5. N. R. Barton and V. Choubey, "The shear strength of rock joints in theory and practice," *Rock Mech.*, **10**, 1–54 (1977).
6. K. Bock (ed.), *Introduction to the Mechanics of Jointed Rock* [Russian translation], Mir, Moscow (1983).
7. A. F. Revuzhenko and E. I. Shemyakin, "Formulations of the boundary-value problems of L-plasticity," *Prikl. Mekh. Tekh. Fiz.*, No. 2, 128–137 (1979).
8. A. F. Revuzhenko, "Variational formulations of boundary-value fracture problems," *Prikl. Mekh. Tekh. Fiz.*, No. 4, 148–156 (1980).
9. A. A. Baraykh and A. S. Gegin, "Evaluation of the stability of interchamber pillars," *Fiz. Tekh. Probl. Razrab. Polezn. Iskop.*, No. 1, 30–38 (1997).
10. L. Jing, O. Stephansson, and E. Nordlund, "Study of rock joints under cyclic loading conditions," *Rock Mech. Rock Eng.*, **26**, No. 3, 215–232 (1993).
11. W. Wittke, *Rock Mechanics*, Springer-Verlag, Berlin (1990).
12. S. Bandis, A. C. Lumsden, and N. A. Barton, "Experimental studies of scale effects on the shear behavior of rock joints," *Int. J. Rock Mech., Min. Sci. Geomech. Abstr.*, **18**, No. 1, 1–21 (1981).